Turbulence Spectra and Local Similarity Scaling in a Strongly Stratified Oceanic Bottom Boundary Layer

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Abstract.

In the turbulence inertial subrange, wavenumber spectra of vertical velocity and streamwise velocity in a strongly stratified oceanic bottom boundary layer exhibit the same local similarity scaling laws as found in the stable atmospheric boundary layer. At scales greater than the turbulence inertial subrange, oceanic velocity spectra exceed the universal spectra. The additional energy at large scales is likely contributed by internal waves. If a spectral gap does not exist, there is no effective way to separate internal waves and turbulence in the stratified boundary layer. We propose that the energy containing eddies in the stratified boundary layer have a scale close to the Ozmidov scale, \( \varepsilon^{1/2} N^{-3/2} \) where \( \varepsilon \) is the turbulence kinetic energy dissipation rate and \( N \) is the buoyancy frequency. This leads to a turbulent scaling \( \varepsilon = \beta N \sigma_w^2 \), where \( \beta \) is a dimensionless constant about 0.5–1, and \( \sigma_w^2 \) is the vertical velocity variance. This scaling law of \( \varepsilon \) has been found in free shear flows in the ocean and atmosphere. The turbulence Reynolds stress is related to the vertical velocity variance as \( -\langle u'w' \rangle = 1/4 \sigma_w^2 \). A possible scheme of separating internal waves and turbulence in stratified flows based on the potential vorticity is discussed.

Keywords: vorticity, stratified turbulence, internal waves, spectra

1. Introduction

Within turbulent boundary layers, turbulent fluxes are dynamically important. In numerical models, turbulent fluxes need to be parameterized with numerical schemes based on empirical formulas or theoretical arguments and, sometimes, with delicate tuning. Empirical formulas of turbulent fluxes are determined by combining field observations, laboratory experiments, and theoretical arguments and assumptions.

Since the 1950s, it has been recognized that scaling laws usefully describe atmospheric boundary layers. The famous Monin-Obukhov (MO) similarity-scaling theory was first proposed by Obukhov (1946) and confirmed with measurements by Monin and Obukhov (1954). The MO similarity theory suggests that turbulence and mean properties within the surface boundary layer are determined by surface stress, surface buoyancy flux, and the distance from the boundary. Businger et al. (1971), based on MO scaling, derived empirical formulas for the

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temperature and wind gradients under unstable, neutral, and stable conditions using simultaneous measurements of surface fluxes and profiles of temperature and wind. These empirical formulas have been widely adopted in establishing parameterization schemes in numerical models. They are applicable strictly in the surface boundary layer where the turbulent stress and fluxes are nearly constant.

Mahrt (1999) provided an ideal view of the stable turbulent boundary layer which consists of the roughness sublayer, surface layer, local-similarity layer, and z-less layer (Fig. 1). Heights of these layers are modified by the background stability. Above the surface layer, the MO similarity scaling on the basis of surface fluxes fails because turbulent fluxes generated by other processes, e.g., breaking internal waves, may be equally important as surface fluxes. Nieuwstadt (1984) proposed a local MO similarity scaling in which mean and turbulent properties are scaled by the local turbulent fluxes. The z-less layer is an extension of the local scaling layer for $z \gg \text{the local Monin-Obukhov length scale } L$.

Wyngaard and Kosovic (1994) argued that turbulence properties in the stable boundary layer are variable due to unsteadiness, baroclinicity, terrain slope, and breaking internal gravity waves. Generally it is uncertain whether the local similarity scaling should apply in the intermediate to strongly stratified turbulent boundary layer.

Compared to the atmospheric counterpart, the oceanic turbulent boundary layer is not well studied because it is much harder to make oceanic measurements. The oceanic surface boundary layer is similar to the atmospheric surface boundary layer, but not identical, because surface gravity waves and coupled processes between turbulence and waves, such as Langmuir circulation, are often present (Fig. 1). Turbulence generated by breaking surface gravity waves is clearly not accounted for in the MO similarity scaling (Anis and Moum, 1995).

Under the neutral condition, Sanford and Lien (1999) and Lien and Sanford (2000) confirmed the MO similarity scaling in an unstratified oceanic bottom boundary layer. In the stratified condition, the surface buoyancy flux is important in the atmospheric surface boundary layer and is the primary parameter of the MO similarity scaling. In contrast, the oceanic bottom boundary layer lacks surface buoyancy flux (Fig. 1). Obviously, the MO similarity scaling breaks down in the stable oceanic bottom boundary layer. One may suspect the local similarity scaling (Nieuwstadt, 1984) to be applicable above the surface layer of the oceanic bottom boundary layer. To our knowledge, there has been no documented evidence of the local similarity scaling found in stratified oceanic bottom boundary layer.

Turbulence closure schemes, e.g., Mellor and Yamada, (1982), used in contemporary numerical models, e.g., Davies and Xing (2000), re-
lied heavily on observations taken in the atmospheric boundary layer. Because the oceanic bottom boundary lacks interfacial buoyancy flux, applying these turbulence closure models, derived on the basis of atmospheric observations, on the oceanic bottom boundary layer are inappropriate.

Here, our primary objectives are to examine turbulence velocity spectral properties and similarity scaling laws in a strongly stratified oceanic bottom boundary layer using measurements taken in a tidal channel. In the following section, we will describe experiments and measurements. In section 3, we will briefly review the model spectrum obtained in the stable atmospheric boundary layer and present velocity spectra observed in the stratified oceanic boundary layer. Similarity scalings of the turbulence kinetic energy dissipation rate and the momentum flux are discussed in the section 4. In section 5, we will discuss the spectral gap between internal waves and turbulence, and a possible strategy to separate internal waves and turbulence using potential vorticity.

2. Experiments and Measurements

Data described in this paper were taken in the turbulent boundary layer of a tidal channel, Pickering Passage, WA, during spring ebb tides. The averaged water depth at the experiment site is $\sim 22.5$ m, decreasing from 25 m to 20 m during the ebb tide. A detailed description of the experiment site and bathymetry were presented in Sanford and Lien (1999). Five experiments were conducted between 1993 and 1998 denoted as PP1–5. Four experiments were at the same location within $\sim 150$ m, and one was conducted $\sim 300$-m downstream around a $\sim 4$-m high ridge. Measurements were taken by sensors mounted on the electro-magnetic vorticity meter (EMVM) towed-body operated from the anchored R/V Miller of APL/UW. The EMVM towed-body could be either vertically profiled through the water column or held at fixed depths. The primary sensor mounted on the towed-body was the vorticity meter (VM) which measured turbulence velocity and vorticity using the principle of motional induction. Other sensors mounted on the EMVM towed-body included CTD, shear probe for measuring turbulence kinetic energy dissipation rate, optical backscattering sensor (OBS), and acoustic Doppler velocimeter (ADV). The details of the sensor and measurements were described in Sanford et al. (1999).

The VM sensor mounted on the side of the towed-body measured the vertical velocity, the streamwise velocity, and the spanwise vorticity. The VM sensor mounted on the bottom of the towed-body
measured the spanwise velocity, the streamwise velocity, and the vertical vorticity. A large tail on the EMVM towed-body oriented the vehicle into the dominant streamwise direction. Here, horizontal velocity components are defined according to the orientation of the EMVM towed-body; streamwise velocity $u$ is in the direction of the towed-body, and spanwise velocity $v$ is perpendicular to the towed-body.

The maximum ebb tidal current was $0.6–0.8 \text{ m s}^{-1}$. The background stratification and shear varied greatly during the five experiments (Fig. 2). The water was weakly stratified and sometimes nearly homogeneous in PP1 and PP2, and moderately to strongly stratified in PP3–5. Data taken in PP2 while the water was nearly homogeneous during the peak ebb tide were used to study homogeneous turbulent boundary layer by Sanford and Lien (1999) and Lien and Sanford (2000).

Vertical profiles of shear square $S^2$ showed an order of magnitude increase near the bottom few meters. Above the bottom high-shear layer, the shear decreased in weakly stratified water and remained nearly constant in the moderately to strongly stratified water. These vertical profiles were averaged over $\sim \frac{1}{2}$ ebb tidal period centered on the peak ebb tidal current. As will be shown later, there were significant temporal variations of stratification and shear. Also, small vertical-scale fluctuations have been smoothed by the temporal and vertical spatial averaging procedures.

Vertical profiles of Richardson number $Ri = N^2 / S^2$ are always close to or less than $1/4$. During moderately and strongly stratified conditions, the $Ri$ is within a factor of two from the critical value $1/4$ above 3 mab (meters above bottom). This is consistent with a prediction of constant $Ri$ by the local MO similarity scaling in the $z$-less regime (Nieuwstadt, 1984). During nearly homogeneous conditions, $Ri$ is much less than $1/4$.

In the present analysis, measurements taken at fixed depths will be used to study turbulence spectral properties and similarity scalings. Measurements taken while the EMVM was vertically profiled provide information on shear and stratification.

3. **Spectral Properties in Stable Boundary Layer**

Extensive studies of turbulence spectral properties have been conducted in the atmospheric boundary layer, e.g., the 1968 Kansas experiment and the 1973 Minnesota experiment (Kaimal and Wyngaard, 1990). An understanding of the spectral details of the turbulence is vital to improve parameterizations of turbulent fluxes in numerical models.
In the neutral condition, spectral properties of the oceanic bottom boundary layer are consistent with those found in the atmospheric surface boundary layer (Lien and Sanford, 2000). In stratified flows, both internal waves and turbulence exist in the same physical and Fourier spaces. The turbulence mixing is suppressed and internal waves are enhanced by the stratification. Therefore, velocity spectral properties are significantly altered by the stratification.

We will use turbulence spectra summarized from previous studies of the atmospheric boundary layer, following Kaimal and Finnigan (1994), as a guide for our analysis. In the inertial subrange, where the turbulence scale is much smaller than the scale of energy-containing eddies and much greater than the scale of viscous dissipation, velocity spectra follow the the Kolmogorov scaling, i.e.,

\[ \Phi_q(k_x) = c_q \varepsilon^{2/3} k_x^{-5/3}, \]

where \( \Phi_q \) is the velocity spectrum, \( q = u, v, w \) is the velocity component, \( k_x \) is the streamwise wavenumber, \( \alpha = 1.5 \) is the Kolmogorov constant (Sreenivasan, 1995), \( c_q = 0.33 \) for streamwise velocity, \( c_q = 0.44 \) for the spanwise and vertical velocity, and \( \varepsilon \) is the dissipation rate of turbulence kinetic energy. The spectral level in the inertial subrange is proportional to \( \varepsilon^{2/3} \) which varies with the background forcing and stratification.

At scales larger than the inertial subrange, turbulence spectra cannot be determined by the theoretical similarity scaling. Kaimal (1973) compiled turbulence spectra observed in the stable atmospheric boundary layer and found an empirical form

\[ \frac{k \Phi_q(k)}{\sigma_q^2} = \frac{a(k/k_q^0)}{1 + a(k/k_q^0)^{5/3}}, \]

where \( k_q^0 \) is a reference wavenumber. This empirical spectrum \( \Phi_q \) has a \( -5/3 \) spectral slope at \( k \gg k_0 \), the inertial subrange, and is white at low wavenumbers. The constant \( a = 0.164 \) is fixed so that the integration of the spectrum \( \Phi_q \) equals the total variance \( \sigma_q^2 \). The peak of the variance preserving spectrum \( k \Phi_q \) is at \( k_q^m = 3.77 k_q^0 \). Because this empirical spectrum has to obey the Kolmogorov scaling in the inertial subrange (??), it requires

\[ \varepsilon = b_q \sigma_q^3 k_q^0, \]

where \( b_q = (c_q \alpha)^{-3/2} = 2.87, 1.87, \) and \( 1.87 \) for the streamwise, the spanwise, and the vertical velocity, respectively.

In high energetic stratified turbulent flows, D’Asaro and Lien (2000) found a linear relationship between \( \varepsilon \) and the variance of vertical velocity \( \sigma_w^2 \), i.e.,
\[ \varepsilon = C\sigma_w^2N, \]

where the nondimensional constant \( C = 0.3 - 0.6 \). Weinstock (1981) proposed a similar relation with \( C = 0.4 - 0.5 \) in the stratosphere. Combining the vertical component of (??) and (??), we find \( k_{Oz} = (4 - 10)k_w^u \). The Ozmidov wavenumber \( k_{Oz} = \varepsilon^{-1/2}N^{3/2} \) is well-known as the wavenumber of energy containing eddies for stratified turbulence. Kaimal’s empirical spectrum predicts the similar wavenumber for energy containing eddies, i.e., \( k_w^m = 3.77k_w^u = 0.38 - 0.94k_{Oz} \).

In the following, we will present observed oceanic spectra of streamwise velocity and vertical velocity, compare them with (??) in the inertial subrange, normalize the observed spectra taken in a strongly stratified period by their variance, and fit the normalized spectra to (??) to determine \( k_w^u \) and \( k_{Oz}^u \).

3.1. Observed Spectra

Velocity spectra were computed using measurements taken at fixed depths. There was a total of 459 time segments. These measurements were taken mostly close to the bottom (76% below 9 mab) for detailed observations of turbulence in the bottom boundary layer and were often longer than 10 min. Some short segments, generally < 3 min, were taken near the surface.

Typical streamwise and vertical velocity spectra observed during strong ebb tides are shown in Fig. 3. The streamwise velocity spectra exhibit a clear -5/3 spectral shape in the wavenumber range 0.3–4 m\(^{-1}\), slightly more than a decade. At \( k_x < 0.3 \) m\(^{-1}\), streamwise velocity spectra are slightly flatter than a -5/3 slope, and at \( k_x > \sim 4 \) m\(^{-1}\), spectra drop steeper than a -5/3 slope as a result of sensor response (see Sanford et al., 1999). Vertical velocity spectra exhibit a white spectra at \( k_x < 1 \) m\(^{-1}\), a -5/3 spectral slope in 1–4 m\(^{-1}\), and a steeper spectral rolloff at \( k_x > 4 \) m\(^{-1}\) as a result of sensor response. These observed spectral shapes are consistent with Kaimal’s empirical spectrum in the stable boundary layer, although there were taken in various stratification conditions.

To further evaluate the inertial subrange, we compare \( \varepsilon \) measured from the shear probe with \( \varepsilon \) estimated by observed velocity spectra in the inertial subrange. Estimates of \( \varepsilon \) were computed by multiplying the velocity spectra by \( k_w^{5/3} \), determining the inertial subrange (white), and averaging over the selected inertial subrange. The streamwise velocity spectrum generally has its inertial subrange extended to a lower wavenumber than the vertical velocity spectrum does (Fig. 3). Consequently, \( \varepsilon \) estimated from the vertical velocity spectrum is noisier.
than that estimated from the streamwise velocity spectrum (Fig. 4). Nevertheless,good agreement is found between $\varepsilon$ measured from the shear probe and $\varepsilon$ estimated by velocity spectra. The $\varepsilon$ measured from the shear probe is about twice that computed from the velocity spectra. This discrepancy might result from inaccuracy in the calibration of the shear probe.

3.2. Universal Spectral Form?

Kaimal (1973) compiled spectra from the Kansas experiment and found a universal spectral form (??) for turbulent velocity in a stable atmospheric boundary layer. Removing internal gravity waves from observed measurements is not a trivial task. In the analysis of turbulence in a stable atmospheric boundary layer, Nieuwstadt (1984) successfully removed effects of internal waves by 1) choosing measurements taken during periods of strong turbulence, 2) choosing measurements when velocity variances decrease with altitudes, and 3) low-pass filtering of the measurements. The benefit of 1) is obvious. The second scheme assumes the boundary is the source of turbulence. The third procedure assumes that there is a spectral gap between internal waves and turbulence. Support for 2) and 3) were found by Caughey et al. (1979) in a weakly stable boundary layer. In a very stable boundary layer, enhanced turbulence, generated by processes such as breaking internal waves, was sometimes found detached from the boundary (Mahrt, 1999). In this circumstance, the spectral gap between internal waves and turbulence might not exist. Therefore, in a very stable boundary layer it is impossible to completely exclude effects of internal waves from measurements by applying procedures 2) and 3).

In a search for a similar universal spectrum as found by Kaimal (1973) for the oceanic stratified bottom boundary layer, we will use measurements taken in November 19, 1998 during the PP5 experiment. These measurements were taken at fixed depths in a stratified flow for a relatively longer period, nominally at 1.6, 3.6, 5.6, and 7.6 mab, for typically 14 min (Fig. 5). Within each fixed depth segment, there were occasional rapid changes of depth due to winch motion. We label the segments according to the period when the EMVM was relatively steady at a fixed depth (see labels in Fig. 5c). Between fixed-depth segments, the EMVM was profiled for measuring vertical stratification and shear. Estimates of $N^2$ closest to the time of fixed-depth segments were shown in Fig. 5b. The water was nearly homogeneous within the first 1/2 hour, $N^2 \leq 3 \times 10^{-5} \text{s}^{-2}$ for one and half hours, $N^2 \geq 10^{-4} \text{s}^{-2}$ for $\sim$2 hrs when the ebb tidal current increased, and became nearly homogeneous within the
last hour of measurements when the tidal current began to reverse. The tidal speed during the peak tide was \( \sim 0.4 \text{ m s}^{-1} \) at 7.6 mab.

Note that the stratification was highly variable in time and with depths. As an example, three vertical profiles of density and stream-wise velocity taken within 15 min before and after segment 17 are shown in Fig. 6. The first two profiles, taken within 1 min, showed a similar density structure but significantly different velocity structure. The third profile, taken \( \sim 15 \) min later, exhibited a very different density structure. The compass of the EMVM showed that the orientation of the mean flow rotated in depth and varied in time. These profiles were averaged at 1-s intervals, corresponding to 0.1–0.2 m in the vertical with the typical profiling speed of 0.1–0.2 m s\(^{-1}\). Both velocity and density profiles showed a small vertical scale, \(< 1 \) m, fluctuations embedded in the background profiles. Steps of constant density and small-scale overturning are present. Similar complex vertical structures have been observed in the stratified atmospheric boundary layer. Chinomas (1999) suggested that these fine scale steps play crucial roles in triggering shear instability.

In the turbulent boundary layer, turbulence properties are strongly modified by background shear and stratification. Because of the aforementioned complex variability of shear and stratification, we expect that turbulence properties will vary at the similar time scale. The variation of vertical velocity spectra is illustrated with the \( \sim 14\)-minute measurements taken at 7.5 mab, segment 17, when the ebb tidal current reached its maximum value of \( \sim 0.4 \) m s\(^{-1}\) (Fig. 7). During this period, \( \varepsilon \) was \( 2 \times 10^{-7} - 2 \times 10^{-5} \) W kg\(^{-1}\), \( U \) was 0.29 – 0.51 m s\(^{-1}\), and \( w \) was \( \pm 0.06 \) m s\(^{-1}\). The mean vertical velocity variance \( \sigma_w^2 \) was \( 3 \times 10^{-4} \) m\(^2\) s\(^{-2}\), but reached to \( 10^{-3} \) m\(^2\) s\(^{-2}\) occasionally.

We performed a wavelet analysis on the vertical velocity using the Daubechies’ least asymmetric filter of length 8 (Percival and Walden, 2000). The linear phase shift due to the asymmetry of the wavelet filter has been corrected. Most of vertical velocity variance was contributed by eddies of wavenumber \( k_x \approx 1 \) m\(^{-1}\), ranging between 0.5 and 10 m\(^{-1}\) (white shading). The Ozmidov wavenumber \( k_{Oz} = N^3/2 \varepsilon^{-1/2} \) is estimated \( \sim 1.5 \) m\(^{-1}\) giving \( \varepsilon \) of \( \sim 5 \times 10^{-6} \) W kg\(^{-1}\), and \( N^2 \) of \( \sim 5 \times 10^{-4} \) s\(^{-2}\). Vertical velocity spectra varied significantly in time. Examples of two vertical velocity spectra computed by averaging 2-min of wavelet spectra \( \sim 9\)-min apart show that spectral levels change by a factor of two in the inertial subrange, consistent with the change of \( \varepsilon \). Between \( k_x \) of 0.2 and 20 m\(^{-1}\), spectral shapes seem to resemble Kaimal’s model spectrum, white at low wavenumbers and -5/3 in inertial subrange. Below 0.2 m\(^{-1}\), spectral levels elevate presumably due to non-turbulent
motions. The elevated spectral level at $k_x \geq 20 \text{ m}^{-1}$ is likely due to over-correction by the sensor response (Sanford et al., 1999).

Our observed spectra do not show a clear spectral gap between internal waves and turbulence as found in the atmospheric boundary layer (Caughey et al., 1979). To remove effects of internal waves, we examine individual spectrum, carefully exclude low-wavenumber spectra where spectral levels are elevated above the background white spectrum, and exclude high-wavenumber spectra where noise spikes are present. After the above procedure, 4 of 26 vertical velocity spectra are removed. The remaining 22 velocity spectra exhibit a similar spectral shape, but the levels differ by nearly 2 decades (Fig. 8). These spectra are fitted to Kaimal’s model spectrum (2) to obtain $k_{m}^{w}$. The estimates of $k_{m}^{w}$ vary between 0.26 and 0.64 m$^{-1}$ with a mean of 0.48 m$^{-1}$. The corresponding estimates of $k_{m}^{u}$ are 1–2.5 m$^{-1}$.

We normalize observed spectra by their variances and normalize wavenumbers by estimated $k_{m}^{w}$. This normalization procedure reduces the range of spectral levels to a factor of 5. The mean of normalized spectral shape resembles Kaimal’s model spectrum. However, the observed normalized spectrum slightly exceeds the model spectrum below the inertial subrange, $k_x \leq 10 k_{m}^{w}$ and at $k_x \geq 30 k_{m}^{w}$. We suggest that the extra energy below the inertial subrange is due to the combination of internal waves and turbulence. Internal waves and turbulence exist at the same scales. Within these common wavenumbers, it is impossible to remove internal waves from observations on the basis of characteristics of wavenumber spectra. The extra energy at high wavenumbers is due to spurious velocity noise produced during depth changes and cable strumming which was not completely removed by our manually filtering process.

We apply the same analysis to the streamwise velocity spectra. Only 20 of the 26 streamwise velocity spectra exhibit a clear inertial subrange (Fig. 9). Spectral levels varied by ~1.5 decades. Fitting these spectra to Kaimal’s model spectrum yields $k_{0}^{u}$ of 0.03–0.2 m$^{-1}$, with a mean of 0.08 m$^{-1}$. The ratio between the means of $k_{0}^{w}$ and $k_{0}^{u}$ is 1/6, close to the 1/8 found in the stable atmospheric boundary layer (Moraes, 1988). The corresponding wavenumber of the spectral peak is $k_{m}^{u} \approx 0.3 \text{ m}^{-1}$. Because our spectra were computed with measurements about 10-min long, observed streamwise velocity spectra barely resolve wavenumbers below 0.05 m$^{-1}$. Our estimates of $k_{0}^{u}$ are likely overestimated.

The streamwise velocity variance is about four times the vertical velocity variance, close to the ratio of 3 found in the stable atmospheric boundary layer (Kaimal, 1973). The observed higher ratio is possibly due to contributions from internal waves, which are mostly dominated by horizontal velocity variance. Because the spectral peak
of the streamwise velocity is at a wavenumber, \( k_m^u \approx 0.3 \text{ m}^{-1} \), not much greater than the smallest resolvable wavenumber \( \sim 0.05 \text{ m}^{-1} \), it is difficult to identify the white-spectrum regime below \( k_m^u \) and to remove elevated variances at low-wavenumbers contributed by internal waves. Consequently, we believe that observed streamwise velocity spectra, after the filtering procedure, are still significantly contaminated by internal waves.

Normalizing streamwise velocity spectra by variances and wavenumbers by \( k_0^u \) slightly collapses spectra in the inertial subrange, but the spread at the spectral peak is still about one decade. The mean normalized spectrum has a similar spectral level to that of the Kaimal’s model, except near the spectral peak where the observed spectrum is twice that of the Kaimal’s spectrum.

Overall, spectra observed in the stratified oceanic bottom boundary layer show some agreement with Kaimal’s model spectrum in the inertial subrange. However, there are significant departures at wavenumbers below the inertial subrange, which are likely due to the presence of internal waves. Our filtering procedure appears ineffective at removing internal waves. The major obstacle lies in the fact that no clear spectral gap exists between internal waves and turbulence in the oceanic stratified turbulent boundary layer.

### 4. Similarity Scaling

In the very stable atmospheric boundary layer, enhanced turbulence was often found in layers of enhanced shear and reduced Ri with little dynamic connection with the boundary (Finnigan, 1999). In other words, turbulence in the very stable atmospheric boundary layer may resemble free shear stratified turbulence. Both Reynolds stress and buoyancy flux are required to examine the local similarity scaling. Our velocity and CTD sensors are separated by 0.5 m which results in inaccurate estimates of buoyancy flux.

The scaling of turbulence kinetic energy dissipation rate \( \varepsilon \) could be established on the basis of previous studies of shear turbulence. In the z-less stable boundary layer, Nieuwstadt (1984) showed that \( \sigma_w^2 \approx -2\langle u'w' \rangle \) and \( \text{Ri} \approx 1/4 \), i.e., the shear \( d_zU \approx 2N \). Assuming a local balance between the shear production and the turbulence kinetic energy dissipation, i.e., \( \varepsilon \approx -\langle u'w' \rangle d_zU \), one could derive a scaling laws \( \varepsilon \approx \sigma_w^2N \). This scaling is similar to that proposed by Gargett (1984), Weinstock (1981), and D’Asaro and Lien (2000), as mentioned previously.
In the oceanic microscale community, the diapycnal diffusivity $K_{\rho}$ is often computed as $K_{\rho} = 0.2\varepsilon N^{-2}$ (Osborn, 1980). Assuming the turbulent Prandtl number of 1, one deduces a similarly scaling $\varepsilon \approx 0.84\sigma_w^2 N$ on the basis of Nieuwstadt’s (1984) result of $\sigma_w^2 \approx -2\langle u'w' \rangle$ and Osborn’s (1980) expression for diapycnal diffusivity.

Matching Kaimal’s model spectrum with the Kolmogorov similarity scaling in the inertial subrange, one obtains a relation $\varepsilon = 1.87\sigma_w^2 k_{\omega m} = 0.5\sigma_w^2 k_{Oz} = N^{3/2}\varepsilon^{-1/2}$, a similar scaling of $\varepsilon$ is obtained as $\varepsilon = 0.62\sigma_w^2 N$.

The above discussion demonstrates that three independent approaches arrive at a common scaling for $\varepsilon$ in the shear stratified turbulent flows. The Kaimal’s model spectrum is consistent with results of stratified free shear turbulence assuming the Ozmidov scale being the scale of energy containing eddies.

Our observations show a clear correlation between $\varepsilon$ and $\sigma_w^2$ spanning nearly 1.5 decades (Fig. 10). Unfortunately, we do not have simultaneous measurements of $\varepsilon$ and $N$. Therefore, only measurements taken during the period of nearly steady stratification $N_0 \approx 0.02$ s$^{-1}$, segments 11–21, were plotted. This period corresponds to the peak tidal flow and the strongest stratification. In Fig. 10, $\varepsilon$ and $\sigma_w^2$ were computed over 40-s intervals. This time interval was chosen so that it is sufficiently long to capture vertical velocity variance of turbulence and it is short enough to retain the temporal variation of $\varepsilon$ and $\sigma_w^2$. For a typical mean streamwise velocity exceeding 0.3 m s$^{-1}$, the 40-s time interval allows the smallest resolvable wavenumber of 0.5 m$^{-1}$. This is be sufficient to capture most of the vertical velocity variance.

The best fit to the ratio of $\varepsilon$ and $\sigma_w^2$ is 0.03 s$^{-1}$. As shown in Fig. 4, the observed $\varepsilon$ is likely overestimated by a factor of two due to possible calibration inaccuracy of shear probes. Correcting for this factor of two, our observations suggest a relation of $\varepsilon_{corr} \approx 0.75\sigma_w^2 N_0$, where $\varepsilon_{corr} = 0.5\varepsilon$. This supports the suggested scaling for $\varepsilon$.

We also seek the relation between vertical velocity variance and the momentum flux. Again, a 40-s averaging time interval is used. A good correlation between $\sigma_w^2$ and $\langle u'w' \rangle$ is found (Fig. 10). Note that the computed momentum flux $\langle u'w' \rangle$ is occasionally positive, but the mean is negative. The best-fit relation is $\sigma_w^2 \approx -4\langle u'w' \rangle$. The proportionality 4 is greater than Nieuwstadt’s result of 2. This may be due to the contamination by internal waves which was not completely removed from our measurements. Note that observations of Caughey et al. (1979) exhibited a similar higher ratio (Fig. 2 of Nieuwstadt, 1984)).
5. Discussions

5.1. Spectral Gap

Within the stable atmospheric boundary layer, a spectral gap at \( \sim 0.01 \) Hz has often been observed separating low-frequency internal waves and high-frequency turbulence (Caughey et al., 1979). In studying the turbulent boundary layer, it is possible to filter out internal wave energy at frequencies below the spectral gap (e.g., Caughey et al., 1979; and Nieuwstadt, 1984).

Our observed spectra of velocity components do not show a spectral gap separating internal waves and turbulence. Therefore, it is impossible to distinguish between internal waves and turbulence using velocity spectra. (Caughey, 1984) pointed out that at higher levels of the stable boundary layer the spectral gap might disappear because, with the increasing buoyancy frequency, the spectral peak of internal waves moves closer toward the peak of turbulence. In the surface boundary layer, the wavenumber of the spectral peak of turbulence velocity decreases with the altitude and is independent of the buoyancy frequency.

In order to have a discernible spectral gap, internal waves and turbulence must have their spectral peaks at separate wavenumbers, or frequencies, and they must have comparable energy (Fig. 12). As illustrated in Fig. 12, if the energy of internal waves \( E_{iw} \) increases, the spectral peak of turbulence may be overwhelmed by the internal wave spectrum \( \Phi_{iw} \) and becomes indiscernible even though the dominant wavenumbers for internal waves and turbulence, \( k_{iw} \) and \( k_{m} \), are separated. Also the spectral gap becomes indiscernible if the peak of two spectra moves closer, as discussed by Caughey (1984).

In the strongly stratified oceanic bottom boundary layer, the \( k_{iw} \) may depend on the bottom topography, the background shear and buoyancy frequency for shear instability flows, and the water depth. The energy level of \( \Phi_{iw} \) should vary with the shear of the tidal flow and the buoyancy frequency; both are highly variable in time. Our observations taken in PP5 were in a strongly stratified regime where the Ozmidov scale is \( O(1 \) m). These observations were likely in the local-similarity-scaling layer and the z-less layer. In these regimes, the spectral peak \( k_{m} \) is less dependent on the boundary, but on the stratification and the local flux. We suggest that \( k_{m} \) is equivalent to the Ozmidov wavenumber \( k_{Oz} = N^{3/2} \varepsilon^{-1/2} \). In the observed energetic shear-stratified bottom boundary layer, intermittent wave breaking supplies energy to turbulence. Therefore, \( \varepsilon \) as well as \( k_{Oz} \) depend on \( N \), the shear, and the internal-wave energy.
In the observation site, there are broad-crested ripples on the bottom with \( \sim 0.3 \)-m height, \( \sim 16 \)-m wavelength, and \( \sim 100 \)-m crest length. For internal waves generated at the bottom, the dominant wavenumber \( k_{iw} \) is about 0.4 \( m^{-1} \). This is close to the typical Ozmidov wavenumber \( O(1 \ m^{-1}) \) in our observations. This might explain the absence of spectral gap between internal waves and turbulence in our observed velocity spectra.

5.2. SEPARATING INTERNAL WAVES AND TURBULENCE USING POTENTIAL VORTICITY

In the Lagrangian frequency domain, internal waves and turbulence may be separated by the buoyancy frequency \( N \) (D’Asaro and Lien, 2000) The stratified flow can be represented as the superposition of isotropic turbulence at Lagrangian frequencies greater than \( N \) and anisotropic, dominantly horizontal motions, internal waves at Lagrangian frequencies below \( N \). This means that one might separate internal waves and turbulence in the Lagrangian frame.

In the wavenumber or Eulerian frequency domain, separating internal waves and turbulence would be impossible if a spectral gap does not exist, as discussed in the previous section. In flows undergoing strong shear-instability, internal waves and turbulence are coupled, and there is not a clear separation between them. However, there are some distinct characteristics distinguishing internal waves from turbulence. Linear internal waves are governed by a dispersion relation. The energy of internal waves exists dominantly in a specific wavenumber-frequency domain. As the nonlinearity increases, the energy begins to spread about the dispersion curve (Holloway, 1983). In contrast, turbulence does not have a dispersion relation because it is strongly nonlinear. Therefore, one may extract the energy of internal waves following the dispersion curve, and extract energy of turbulence away from the dispersion curve of linear internal waves. Unfortunately, this requires taking time-series measurements over 3-D spatial scales covering the complete internal wave spatial and time scales. This is not feasible with the current technology.

Lien and Müller (1992a) presented consistency tests for spectral properties of internal waves and vortical motions in the ocean. The vortical motion carries the potential vorticity, defined as the inner product of the vorticity vector and the density gradient, and the internal wave has no fluctuating potential vorticity. Turbulence represents the vortical motion at small-scales. It often has strong vortex stretching across the isopycnal surface. Lien and Müller (1992b) presented a scheme to separate the energy of observed small-scale motions into
internal waves and vortical modes. The scheme requires measurements of components of potential vorticity and works only in the linear to weakly nonlinear domain. Nevertheless, measuring the components of potential vorticity may be one of the effective ways to separate internal waves and turbulence.

The VM sensors measured both the vertical and spanwise components of vorticity. Unfortunately, simultaneous density gradients were not measured, so an estimate of the potential vorticity was not possible. We expect isopycnal surfaces to be nearly horizontal at large scales where internal waves dominate. The horizontal component of vorticity has to be much greater than the vertical component so that the perturbation potential vorticity vanishes. Observed spectra of vertical and spanwise vorticity components had nearly identical spectral levels over the entire wavenumber domain. Spectra of vorticity components computed using measurements taken in segment 17 are shown in Fig. 11. Observed vorticity spectra are in good agreement with those of isotropic turbulence after applying the sensor response function. The sensor response function of vorticity measurements taken by the VM sensor has been discussed by Sanford and Lien (1999). Our results suggest the observed vorticity variances are dominated by turbulence with no signatures of internal waves.

6. Summary

We examine turbulence velocity spectra in a stably stratified boundary layer. In a strongly stratified turbulent boundary layer, separation of waves and turbulence remains an unsolved issue. The mean vertical profile of streamwise velocity exhibits log-linear structure. Individual profiles of streamwise velocity and density show very complex vertical structure; small-scale steps, internal waves, and turbulence are present. Results of the wavelet analysis show significant variations of velocity spectra. Low-wavenumbers spectral plateaus are ascribed to internal waves.

After carefully filtering out internal waves at low-wavenumbers and instrumental noise at high-wavenumbers, the observed spectra were normalized by their variances and compared with the canonical spectrum summarized from observations in the Kansas experiment (Kaimal, 1973). Our normalized spectra of vertical and streamwise velocity components agree well with the Kaimal’s model spectrum in the inertial subrange. The reference wavenumber $k_0$ is estimated to be 0.48 m$^{-1}$ and 0.08 m$^{-1}$ for the vertical velocity, and streamwise velocity spectra,
respectively. The ratio between \( k^w_0 \) and \( k^u_0 \) is 6, close to that found in the atmospheric boundary layer.

At subinertial wavenumbers, observed spectra of vertical and streamwise velocity consistently exceed Kaimal’s model spectra. Probable reasons for this discrepancy are (1) the contamination of internal waves, (2) inaccurate estimates of \( k_0 \), and (3) inaccurate estimates of velocity variances. If internal waves and turbulence coexist at scales greater than the inertial subrange, there is no apparent method to separate one from the other in the velocity spectrum. The estimate of \( k_0 \) is sensitive to the spectral shape and variance. Furthermore, the variance of streamwise velocity could be underestimated for some short-record measurements which do not resolve large scale eddies.

We propose that the parameter \( k_m = 3.77k_0 \) in Kaimal’s model spectrum is related to the Ozmidov wavenumber \( k_{Oz} \). Unfortunately, we lack simultaneous estimates of \( N \) and \( \epsilon \) during measurements at fixed depths so that a direct correlation between \( k_m \) and \( k_{Oz} \) cannot be confirmed. We support the idea that turbulence in the stratified boundary layer, above the surface layer, behaves as free shear turbulence.

A relation \( \epsilon \approx 0.75\sigma_w^2N_0 \) was found when the water was strongly stratified. This is consistent with Kaimal’s model spectrum and \( k_m = k_{Oz} \). This scaling has been proposed by previous studies of wall-free shear-driven turbulence (Gargett, 1984; Weinstock, 1981, D’Asaro and Lien, 2000). Our results support the hypothesis that turbulence properties in the very stably stratified boundary layer are analogous to the free shear turbulence (Fig. 1).

A weak correlation was found between momentum fluxes and vertical velocity variances. The proportionality was slightly different from the result of Nieuwstadt (1984). This result also reconciles the scaling of \( \epsilon \), the local balance between shear production and turbulence dissipation, and the critical Richardson number.

The parameterization of turbulence in a strongly stratified turbulent boundary layer is complicated by the existence of internal waves. As concluded by Nappo and Johansson (1999), in the summary of Løvanger international workshop on turbulence and diffusion in the stable planetary boundary layer, separating internal waves and turbulence requires more theoretical and observational studies. There is a hope of separating internal waves and turbulence on the basis of potential vorticity. Therefore, observations of potential vorticity will be needed to improve our understanding and parameterization schemes of turbulence in strongly stratified flows. Future studies should focus on the details and parameterizations of wave/turbulence interactions in the stratified turbulent boundary layer.
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References


Figure 1. Sketch of flow regimes in stable atmospheric boundary layer (after Mahrt, 1999) and oceanic bottom boundary layer. \( Q \) represents the interfacial buoyancy flux and \( \tau \) is the interfacial stress. \( U \) represents a time mean velocity profile.
Figure 2. Vertical profiles of (a) stratification ($N^2$), (b) shear-squared ($S^2$), and (c) gradient Richardson number ($Ri$) averaged over each day of each experiment. Vertical profiles of streamwise velocity and density were averaged in 0.5-m vertical bins. The thick solid curve represents the PP1, thin solid curves represent PP2, the thick dashed curve represents the PP3, thin dashed curves represent PP4, and gray curves represent PP5.
Figure 3. Spectra of (a) streamwise velocity, and (b) vertical velocity observed during PP1, PP2, PP3, PP4, and PP5. Only spectra taken during the strong ebb tide, $U > 0.4$ m s$^{-1}$, are shown. In (c) and (d), spectra are multiplied by $k_x^{5/3}$, where $k_x$ is the streamwise wavenumber, so that the inertial subrange appeared as white for the convenience of estimating the dissipation rate of turbulence kinetic energy. Spectral levels are offset by $10^{-2}$, $10^{-4}$, $10^{-6}$, and $10^{-8}$ for PP2, PP3, PP4, and PP5, respectively. Spectral slopes of -5/3 are indicated in (a) and (b).
Figure 4. Comparison of $\varepsilon$ observed from the shear probe $\varepsilon_{sp}$ and $\varepsilon_{\Phi_u}$ estimated from the observed spectra of streamwise velocity (circles), and vertical velocity ($X_s$). Vertical and horizontal lines represent 95% confidence intervals for the estimated and observed $\varepsilon$, respectively.
Figure 5. Time series of (a) streamwise velocity, (b) buoyancy frequency-squared $N^2$, and (c) altitude (distance above the bottom) of the EMVM in Nov. 19, 1998 during PP5. The $N^2$ was computed using the profiling data. $N^2$ values before and after measurements at fixed depths are shown. Open circles indicate negative $N^2$. In panel (c), labels denote the segment numbers.
Figure 6. Vertical profiles of (a) density, (b) streamwise velocity, and (c) compass of the EMVM taken before (profiles 1 and 2) and after (profile 3) segment 17 (see Fig. 5).
Figure 7. Time series of (a) $\varepsilon$, (b) streamwise velocity $U$, (c) vertical velocity $w$, and (d) variance of $w$ averaged in 1-sec interval during the segment 17. The gray-scale shading in (e) shows the variance preserving plot of vertical velocity as a function of time and wavenumber computed by wavelet analysis. Two horizontal thick bars at the bottom of (e) mark the two time periods when the vertical velocity spectra shown in (f) were computed using results from the wavelet analysis. The gray curve in (f) represents Kaimal’s model spectrum (eq. 2) assuming $\sigma_w^2 = 2 \times 10^{-4}$ m$^2$ s$^{-2}$, and $k_0^w = 0.5$ m$^{-1}$. 
Figure 8. (a) Observed vertical velocity spectra and (b) normalized vertical velocity spectra (dots). The two thick solid curves in (a) show two spectra with significantly different spectral levels. The thick solid curve in (b) shows the mean of the observed normalized spectra and the thick gray curve is Kaimal's model spectrum (Kaimal, 1973).
Figure 9. (a) Observed streamwise velocity spectra and (b) normalized streamwise velocity spectra (dots). The two thick solid curves in (a) show two spectra with significantly different spectral levels. The thick solid curve in (b) shows the mean of the observed normalized spectra and the thick gray curve shows the model spectrum presented by Kaimal (1973).

Figure 10. Scatter plots of (a) turbulence kinetic energy dissipation rate $\epsilon$ vs. variance of vertical velocity $\sigma_w^2$, and (b) momentum flux $\langle u' w' \rangle$ vs. $\sigma_w^2$. Solid dots are average values of $\epsilon$ and $\langle u' w' \rangle$ in grid bins of $\sigma_w^2$. Dashed lines represent curves of the best fit.
Figure 11. Comparison of spectra of observed vertical vorticity (black solid curve) and spanwise vorticity (dashed curve) computed from measurements taken at the segment 17 (see Fig. 5). The gray curve is the theoretical vorticity spectrum of isotropic turbulence applying the sensor response function of the VM sensor.
Figure 12. Diagram illustrating spectral gap between internal waves and turbulence. Three thin curves represent velocity spectra of internal waves $\Phi_{iw}$ and the thick curve represents the velocity spectrum of turbulence $\Phi_\varepsilon$. The wavenumber of the peak of $\Phi_{iw}$ is denoted as $k_{iw}$ and the wavenumber of the peak of $\Phi_\varepsilon$ is denoted as $k_m$. 